

## Fluid flow in micro-channels

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### Abstract

We consider the problem of liquid and gas flow in micro-channels under conditions of a small Knudsen and Mach numbers, that correspond to continuum model. Data from the literature on pressure drop in circular, rectangle, triangular and trapezoidal micro-channels with hydrodynamic diameter ranging from 1.01  $\mu\text{m}$  to 4010  $\mu\text{m}$  are analyzed. The Reynolds number at transition from laminar to turbulent flow is considered. Attention is paid to comparison between predictions of the conventional theory and experimental data, obtained during the last decade, as well as to discussion of possible sources of unexpected effects which were revealed by a number of previous investigations.  
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### 1. Introduction

Flow in small tubes has been studied by a many researches over the years. Schlichting [1] documents theories and experimental data from the pioneering works by Hagen [2] and Poiseuille [3] until 1979. Rapid development of micro-mechanics, stimulated during the last decades numerous investigations in the field of fluid mechanics of micro-devices (Ho and Tai [4], Gad-el-Hak [5]). Research in this field is important for different applications in micro-system technology, in particular, micro-scaled cooling systems of electronic devices which generate high power (Tuckerman [6], Incropera [7]).

The problems of micro-hydrodynamics were considered in different contexts: (i) drag in micro-channels with

hydraulic diameter from  $10^{-6}$  m to  $10^{-3}$  m at laminar, transient and turbulent single-phase flows, (ii) heat transfer of liquid and gas flows in small channels, (iii) two-phase flow in adiabatic and heated micro-channels. The studies performed in these directions encompass a vast class of problems related to flow of incompressible and compressible fluids in regular and irregular micro-channels under adiabatic conditions, heat transfer, as well as phase change.

In spite of the existence of numerous experimental and theoretical investigations, a number of principal problems related to micro-fluid hydrodynamics are not well studied. There are contradictory data on drag in micro-channels, transition from laminar to turbulent flow etc. That leads to difficulties in understanding the essence of this phenomenon and is a basis for questionable discoveries of special “micro-effects” [8,4,9–12]. The latter were revealed by comparison of experimental data with predictions of conventional theory based on the Navier–Stokes equations. The discrepancy between

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**Nomenclature**

$\bar{u}$	average velocity
$v_*$	friction velocity
$\ell$	the length of the micro-channel
$r_0$	the radius of the micro-channel
$\Delta P$	pressure drop
$c_p$	specific heat
$u_k$	velocity at the top of the roughness element
$d$	diameter
$E$	activation energy
$h$	side of an equilateral triangle; enthalpy
$H$	side of a rectangle; depth of plane micro-channel
$k$	thermal conductivity
$Kn$	Knudsen number
$k_s$	height of roughness
$M$	Mach number
$m$	mass flow rate
$P$	pressure
$Po$	Poiseuille number
$R$	universal gas constant
$Re$	Reynolds number
$T$	temperature
$t$	time
$u$	velocity
$x, y, z$	Cartesian coordinates

*Greek symbols*

$\alpha, \beta$	semi-axis of the ellipse
$\eta$	dimensionless radius
$\theta$	dimensionless temperature
$\lambda$	drag coefficient
$\mu$	dynamic viscosity
$\nu$	kinematic viscosity
$\zeta$	height to width ratio
$\rho$	density
$\tau$	shearing stress at a wall
$\Phi$	energy dissipation

*Subscripts*

*	characteristic parameter
ac	actual
av	average
cr	critical
ef	effective
exp	experimental
h	hydraulic
in	inlet
m	maximum
out	outlet
theor	theoretical
w	wall

these data was interpreted as a display of new effects of flow in micro-channels. It should be noted that actual conditions of several experiments were often not identical to conditions that were used in the theoretical models. Because of this the analysis of sources of disparity between the theory and experiment is of significance.

We attempt to reveal the actual reasons of disparity between the theoretical predictions and data of measurements obtained for single-phase flow in micro-channels. For this we consider the influence of different factors (roughness, energy dissipation, etc.) on flow characteristics. Some of these were also discussed by Sharp et al. [13,14].

The paper consists of the following: in Section 2 the common characteristics of experiments are discussed. Conditions that are needed for correct comparison of experimental and theoretical results are formulated in Section 3. In Section 4 the data of flow of incompressible fluids in smooth and rough micro-channels are discussed. Section 5 deals with gas flows. The data on transition from laminar to turbulent flow are presented in Section 6. Effect of measurement accuracy is estimated in Section 7. The discussion of the flow in capillary tubes is contained in Section 8.

## 2. Characteristics of experiments

For the analysis of flow in micro-channels we use the experimental data by: (i) Li et al. [15], Yang et al. [16], Pfund et al. [17], Xu et al. [18], Wu and Cheng [19], Maynes and Webb [20], Judy et al. [21], Sharp and Adrian [14] related to flow in smooth micro-channels; (ii) by Peng and Peterson [22], Peng and Wang [23], Mala and Li [24], Qu et al. [25], Pfund et al. [17], Li et al. [15], Kandlikar et al. [26] related to flow in micro-channels with roughness, (iii) by Peng and Peterson [22], Peng and Wang [23], Pfund et al. [17], Li et al. [15] and Sharp and Adrian [14] related to transition from laminar to turbulent flow, (iv) by Harley et al. [27], Hsieh et al. [28] related to gas flow in micro-channels.

Brief characteristics of these experiments are given in Table 1. The flow characteristics (flow rate, pressure gradient, average and fluctuating velocities) were measured for flow in micro-channels, both smooth and with roughness, of different geometry. The physical properties of the fluids used in the experiments, as well as the material of the micro-channels' walls varied widely, which allows analyzing the effect of fluid composition, in particular, ionic component, as well as the influence

Table 1  
Experimental conditions [14–28]

Geometry of the tube	Size		$\frac{L}{L_c}$	Wall		Fluid	$Re$	$u$ (m/s)	Measured parameter
	$d_h$ ( $\mu\text{m}$ )	$h$ ( $\mu\text{m}$ )		Material	Surface				
Circular	$3-4 \times 10^3$	0.51–250	$90-15 \times 10^4$	Glass Silica	Smooth	Liquid	$10^{-3}-4 \times 10^3$	$10^{-4}-30$	Flow rate
Rectangular				Stainless Steel	Roughness	Tap water, distilled water, deionized water, R-134a, methanol, isopropanol, Carbon tetrachloride solutions with several ionic composition	Air $N_2$ He Ar		pressure gradient mean velocity rms of velocity fluctuations
Trapezoidal									
Triangular									

of fluid–wall interaction on the flow characteristics. The Reynolds number in the experiments varied in the range  $10^{-3} < Re < 4 \times 10^3$  (where  $Re = \frac{ud}{\nu}$ , where  $u$  is the velocity,  $d$  is the hydraulic diameter and  $\nu$  is the kinematic viscosity) which covers the regimes of laminar, transition and turbulent flow. The extremely large relative length of micro-channels is characteristic for considered experiments:  $10^2 < \ell/\ell_* < 15 \times 10^4$ , where  $\ell$  is the length of micro-channel,  $\ell_*$  is characteristic length: diameter or depth for circular or rectangular micro-channels, respectively.

In general, conventional theory has been tested for flow in micro-channels by comparing the experimental and theoretical data on pressure drop as a function of flow rate. During the last few years better methods have been used for measurement of the mean velocity, as well as rms of the velocity fluctuations [20,14].

### 3. On comparison between experimental and theoretical results

The classical solution of the problem of steady laminar flow in straight ducts is based on a number of assumptions on flow conditions. They are:

- (i) The flow is generated by a force due to a static pressure in the fluid.
- (ii) The flow is stationary and fully developed, i.e. it is strictly axial.
- (iii) The flow is laminar.
- (iv) The Knudsen number is small enough so that the fluid is a continuous medium.
- (v) There is no slip at the wall.
- (vi) The fluids are incompressible Newtonian fluids with constant viscosity.
- (vii) There is no heat transfer to/from the ambient medium.
- (viii) The energy dissipation is negligible.
- (ix) There is no fluid/wall interaction (except purely viscous).
- (x) The walls are straight.
- (xi) The micro-channel walls are smooth.

In this case the problem of developed laminar flow in a straight duct reduces to integrating the equation

$$\mu \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \frac{dP}{dx} \tag{1}$$

with no-slip condition on the contour, which determine the micro-channel shape.

In Eq. (1)  $u = u(y, z)$  is the longitudinal component of velocity,  $P$  is the pressure,  $\mu$  is the dynamic viscosity,  $x, y, z$  are the Cartesian coordinates,  $x$  is directed along the micro-channel axis.

Table 2  
Parameters of micro-channels [29]

Shape of micro-channel	Characteristic size	Constant in Eqs. (2) and (3)
Circular	$d_* = d$	64
Elliptical	$d_* = \frac{1}{2\sqrt{2}} \frac{2\beta}{(\alpha^2 + \beta^2)^{1/2}}$	64
Equilateral triangular	$d_* = h$	160
Rectangular	$d_* = H$	$128/f(\xi)$

The solution of Eq. (1) leads to the following expression for drag coefficient

$$\lambda = \frac{\text{const}}{Re} \tag{2}$$

or

$$Po = \text{const} \tag{3}$$

where  $\lambda = \frac{2\Delta P}{L} \frac{d_*}{\rho u_*^2}$  is the drag coefficient,  $Po = \lambda \cdot Re$  is the Poiseuille number,  $u_*$  and  $d_*$  are characteristic velocity of the fluid and the micro-channel size, respectively,  $\Delta P$  is the pressure drop on a length  $L$ .

The constant in Eqs. (2) and (3) is determined by the micro-channel shape and does not depend on the flow parameters (Table 2) [29]. In Table 2  $d$  is the diameter of the tube,  $\alpha$  and  $\beta$  are the semi-axes of the ellipse,  $h$  is the side of the equilateral triangle,  $H$  is the side of the rectangle,  $f(\xi)$  is a tabulated function of height to width ratio;  $f(\xi)$  changes from 2.253 to 5.333 when  $\xi$  varies from 1 to  $\infty$ .

The data on pressure drop in irregular channels are presented by Shah and London [30] and White [31]. Analytical solutions for drag of micro-channels with a wide variety of shapes of the duct cross section were obtained by Ma and Peterson [32]. Numerical values of the Poiseuille number for irregular micro-channels are tabulated by Sharp et al. [13]. It is possible to formulate the general features of Poiseuille’s flow, as follows

$$\frac{dPo}{dRe} = 0 \tag{4}$$

$$C^* = \frac{Po_{\text{exp}}}{Po_{\text{theor}}} \tag{5}$$

where  $Po_{\text{exp}}$  and  $Po_{\text{theor}}$  are experimental and theoretical Poiseuille number, respectively.

Eq. (4) reflects the dependence of the drag coefficient on the Reynolds number whereas Eq. (5) shows conformity between actual and calculated shapes of a micro-channel. Condition (5) is the most general since it testifies to identical form of the dependences of the experimental and theoretical drag coefficient on the Reynolds number.

Basically, there may be three reasons for the inconsistency between the theoretical and experimental friction factors: (i) discrepancy between the actual conditions of a given experiment and the assumptions used in

deriving the theoretical value, (ii) error in measurements, (iii) effects due to decreasing the characteristic scale of the problem, which leads to changing correlation between the mass and surface forces [4].

#### 4. Flow of incompressible fluid

##### 4.1. Smooth micro-channels

We begin the comparison of experimental data with predictions of conventional theory for results related to flow of incompressible fluids in smooth micro-channels. In liquid flow in the channels with hydraulic diameter ranging from  $10^{-6}$  m to  $10^{-3}$  m the Knudsen number is much less than unity. Under these conditions, one might expect a fairly good agreement between the theoretical and experimental results. On the other hand, the existence of discrepancy between those results can be treated as display of specific features of flow, which was not accounted for by the conventional theory. Bearing in mind these circumstances, we consider such experiments, which were performed under conditions close to those used for theoretical description of flows in circular, rectangular, and trapezoidal micro-channels.

Glass and silicon tubes with diameters of 79.9–166.3  $\mu\text{m}$ , and 100.25–205.3  $\mu\text{m}$ , respectively, were employed by Li et al. [15] to study the characteristics of friction factors for deionized water flow in micro-tubes in the range of  $Re$  from 350 to 2300. Fig. 1 shows that for fully developed water flow in smooth glass and silicon micro-tubes, the Poiseuille number remained approximately 64, which is consistent with the results in macro-tubes. The Reynolds number corresponding to transition from laminar to turbulent flow was  $Re = 1700$ – $2000$ .

Yang et al. [16] obtained the friction characteristics for air, water, and liquid refrigerant R-134a in tubes with inside diameters from 173 to 4010  $\mu\text{m}$ . The test re-

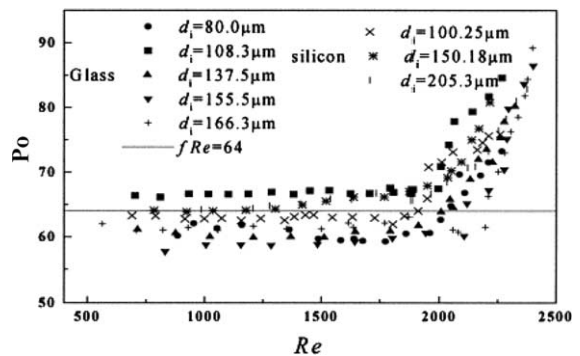


Fig. 1. Dependence of the Poiseuille number on the Reynolds number [15].

sults showed that the pressure drop correlations for large tubes may be adequately used for water, refrigerant, and low-speed air flow in micro-tubes. The laminar-turbulent transition Reynolds number varied from 1200 to 3800 and increased with decreasing tube diameters. The test friction factors agree very well with Poiseuille equation for laminar flow regime.

Pfund et al. [17] studied the friction factor and Poiseuille number for 128–521  $\mu\text{m}$  deep rectangular channels with smooth bottom plate. Water moved in the channels at  $Re = 60$ –3450. In all cases corresponding to  $Re < 2000$  the friction factor was inversely proportional to the Reynolds number. A deviation of Poiseuille number from the value corresponding to theoretical prediction was observed. The deviation increases with a decrease in the channel depth. The ratio of experimental to theoretical Poiseuille number was  $1.08 \pm 0.06$  and  $1.12 \pm 0.12$  for micro-channels with depth 531 and 263  $\mu\text{m}$ , respectively.

Xu et al. [18] investigated deionized water flow in micro-channels with hydraulic diameter ranging from 30  $\mu\text{m}$  to 344  $\mu\text{m}$  at Reynolds numbers ranging from 20 to 4000. Two test modules were used. The first test module consisted of a cover and an aluminum plate, into which a micro-channel, inlet and outlet sumps were machined. A Plexiglass plate was used to cover the channel. The second one was fabricated from a silicon wafer, and a 5 mm thick Pyrex glass was utilized to cover the channel by using anodic bonding. Both of the experimental data obtained in the two test modules showed that the trend of water flow in micro-channels is similar to prediction of the conventional theory as a whole, i.e. that  $Po = \text{const}$  for flow in laminar region. However, some discrepancy between the results obtained in these modules occurred. For flow in the first test module, in which the hydraulic diameter of the channels varied from 50  $\mu\text{m}$  to 300  $\mu\text{m}$  the  $Po$  values in the channels smaller than 100  $\mu\text{m}$  were smaller than that from the prediction of the theory. This phenomenon did not occur in the second test module. Experimental results almost agree with the theory for flow in micro-channels varying from 30  $\mu\text{m}$  to 60  $\mu\text{m}$ . The authors explained the disagreement and showed that the actual dimension of the first module would be inaccurate when the cover and micro-channel plate were sealed by bonding with glue due to the thickness of the layer of the cured material.

The transition to turbulent flow occurred at  $Re$  about 1500. The authors noted that for smaller micro-channels, the flow transition would occur at lower  $Re$ . The early transition phenomenon might be affected by surface roughness and other factors.

Wu and Cheng [19] measured the friction factor of laminar flow of deionized water in smooth silicon micro-channels of trapezoidal cross-section with hydraulic diameters in the range of 25.9–291.0  $\mu\text{m}$ . The experimental data were found to be in agreement within  $\pm 11\%$

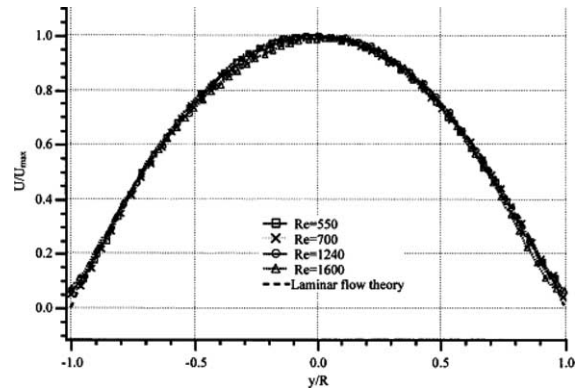


Fig. 2. Velocity profiles versus  $y/R$  [20].

with an existing analytical solution for an incompressible, fully developed, laminar flow under the no-slip boundary condition. It is confirmed that Navier–Stokes equations are still valid for the laminar flow of deionized water in smooth micro-channels having hydraulic diameter as small as 25.9  $\mu\text{m}$ . For smooth channels with larger hydraulic diameters of 103.4–291.0  $\mu\text{m}$ , transition from laminar to turbulent flow occurred at  $Re = 1500$ –2000.

Maynes and Webb [20] presented pressure drop, velocity and rms profile data for water flowing in a tube of diameter 0.705 mm in the range of Reynolds number  $Re = 500$ –5000. The velocity distribution in a cross-section of the tube was obtained using molecular tagging velocimetry technique. The profiles for  $Re = 550, 700, 1240, 1600$  showed excellent agreement with laminar flow theory (Fig. 2). The profiles showed transitional behavior at  $Re > 2100$ . In the range  $Re = 550$ –2100 the Poiseuille number was  $Po = 64$ .

Lelea et al. [33] investigated experimentally fluid flow in stainless steel micro-tubes with diameter of 100–500  $\mu\text{m}$  at  $Re = 50$ –800. The obtained results for the Poiseuille number are in good agreement with the conventional theoretical value  $Po = 64$ . Early transition from laminar to turbulent flow was not observed within the studied range of Reynolds numbers.

Papautsky et al. [34] investigated the flow friction characteristics of water flowing through rectangular micro-channels with width varying from 150  $\mu\text{m}$  to 600  $\mu\text{m}$ , height ranging from 22.71  $\mu\text{m}$  to 26.35  $\mu\text{m}$  and relative roughness of 0.00028. The experiments were conducted in the range of extremely low Reynolds numbers  $0.001 < Re < 10$ . The measurements showed that the ratio  $C^*$  was independent of the  $Re$ . This value was about 1.2 i.e. nearly 20% above the value of  $C^*$  corresponding to conventional theory.

The frictional pressure drop for liquid flows through micro-channels with diameter ranging from 15  $\mu\text{m}$  to 150  $\mu\text{m}$  was explored by Judy et al. [21]. Micro-channels

fabricated from fused silica and stainless were used in these experiments. The measurements were performed with a wide variation of micro-channel diameter, length, types of working fluid (distilled water, methanol, isopropanol) and showed that there are no deviations between the predictions of conventional theory and the experiment. Sharp and Adrian [14] studied the fluid flow through micro-channels with diameter ranging from 50 μm to 247 μm and Reynolds number from 20 to 2300. The measurements agree fairly well with theoretical data.

Cui et al. [35] studied the flow characteristics in micro-tubes driven by high pressure ranging from 1 to 30 MPa. The diameters of the micro-tubes were from 3 to 10 μm and deionized water, isopropanol and carbon tetrachloride were used as the working fluid. The Reynolds number ranged from 0.1 to 24. The measurements showed that the ratio  $C^* = \frac{Po_{exp}}{Po_{theor}}$  varied slightly with the pressure for deionized water. But for the other two liquids, isopropanol and carbon tetrachloride,  $C^*$  increased markedly with pressure (Fig. 3). It should be noted that the viscosity is extremely sensitive to the temperature. The viscosity of water in the pressure range from 1 to 30 MPa can be approximately regarded as constant. Therefore  $C^*$  varied very slightly with pressure. The results of isopropanol and carbon tetrachloride were corrected taking into account the viscosity–pressure relationship to obtain the revised theoretical flow rate and revised normalized friction coefficient. Two revised results of 10 μm for isopropanol and carbon tetrachloride are shown in Fig. 4. It can be seen that the experimental data agree well with theoretical prediction. For  $d = 5 \mu\text{m}$  and  $3 \mu\text{m}$  micro-tubes, similar tendencies were observed.

The experimental results of single-phase flow in smooth micro-channels are summarized in Table 3.

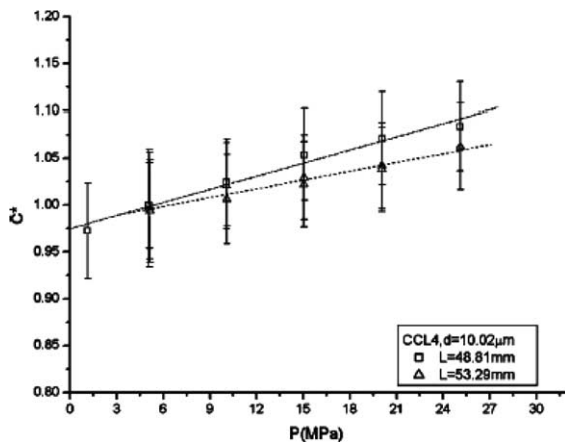


Fig. 3. The normalized Poiseuille number as a function of pressure for carbon tetrachloride in 10 μm micro-tube [35].

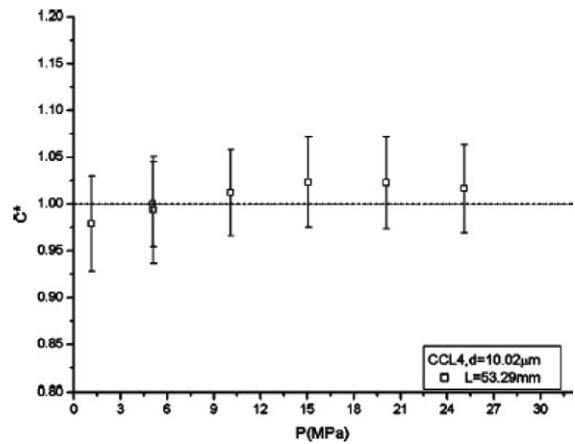


Fig. 4. The revised normalized Poiseuille number as a function of pressure for carbon tetrachloride in 10 μm micro-tube [35].

#### 4.2. Micro-channel with wall roughness

Comparison of experimental data shows that friction characteristics of smooth and rough micro-channels are significantly different. In all cases the roughness leads to increasing friction factor at the same Reynolds number. This effect shows in the range of Reynolds numbers  $10^2 < Re < 3 \times 10^3$ , which corresponds to laminar, transition and turbulent flow regimes.

The existence of roughness leads also to decreasing the value of the critical Reynolds number, at which occurs transition from laminar to turbulent flow. The character of the dependence of the drag coefficient on the Reynolds number in laminar flow remains the same for both smooth and rough micro-channels, i.e.  $\lambda = \frac{const.}{Re}$ .

The general characteristics of experimental investigations of pressure drop in micro-channels with roughness are presented in Table 4. The experiments involve a wide class of flow conditions: shape of micro-channels (circular, rectangular, trapezoidal), their sizes ( $50 \mu\text{m} < d_h < 10^3 \mu\text{m}$ ), and the Reynolds numbers ( $10^2 < Re < 4 \times 10^3$ ). The relative roughness of these micro-channels  $k_s$  varied from 0.32% to 7%. It is known that hydraulically smooth flow regime occurs when the Reynolds number that is defined by height of roughness  $k_s$  and friction velocity  $v_*$  changes in the range  $0 < \frac{k_s v_*}{\nu} < 5$  [36,1], the upper limit of this inequality determines the maximum value of the velocity at which laminar flow is possible. Taking into account that  $v_* = \sqrt{\frac{\tau}{\rho}}$ ,  $\tau = \mu(\frac{du}{dy})_w$ ,  $u = u_m(1 - \eta^2)$ ,  $\eta = \frac{r}{r_0}$  and  $u_m = 2\bar{u}$  ( $\tau$  is the shearing stress at a wall,  $u_m$  and  $\bar{u}$  are the maximum and average velocities,  $r_0$  is the micro-channel radius, subscript w refers to wall) we arrive at the following estimate of the relative roughness, corresponding to the boundary that subdivides the flow in smooth and rough channels.

Table 3  
Experimental results of single-phase fluid flow in smooth micro-channels

Author	Microchannel		Fluid	$Re$	Poiseuille number		$Re_{cr}$
	Shape	$d_h$ ( $\mu\text{m}$ )			$\frac{Po_{exp}}{Po_{theor}}$	Dependence on $Re$ ( $Re < Re_{cr}$ )	
Li et al. [15]	Circular tube	79.9–166.3 100.25–205.5	De-ionized water	350–2300	1	Independent	1700–2000
Yang et al. [16]	Circular tube	173, 4010	Water R-134a	1200–3800	1	Independent	1200–3800
Pfund et al. [17]	Rectangular	128–1050	Water	60–3450	1.08–1.12	Independent	1700–2200
Xu et al. [18]	Rectangular	30–344	Clean water	20–4000	1	Independent	1500
Wu and Cheng [19]	Trapezoidal	25.9–291	De-ionized water		$1 \pm 0.11$	Weak dependence	1500–2000
Maynes and Webb [20]	Circular tube	705	Water	500–5000	1	Independent	2200
Lelea et al. [33]	Circular tube	100, 300, 500	Distilled water	50–800	1	Independent	2800
Pupautsky et al. [34]	Rectangular	Width 150–600 High 22.71–26.35	Water	0.001–10	1.195	Independent	–
Sharp and Adrian [14]	Circular tube	50–245	De-ionized water, 1-propanol and 20% weight of glycerol solution	20–400 400–2900	1	Independent	1800–2300
Judy et al. [21]	Circular tube square micro-channel	15–150	Distilled water methanol isopropanol	8–2300	1	Independent	$\sim 2000$
Cui et al. [35]	Circular tube	3–10	De-ionized isopropanol carbon tetrachloride	0.1–24	$\sim 1$ (for revised data)		

$$\frac{k_s}{r_0} < \frac{5}{1.41Re^{1/2}} \quad (6)$$

For  $Re \sim 2 \times 10^3$ , the relative roughness that corresponds to the boundary between the smooth and roughness channels is  $k_s/r_0 \sim 0.08$ . The latter shows that the hydraulic characteristics of the present micro-channels (Table 4) are close to characteristics of smooth micro-channels.

The deviation of the data related to flow in smooth (Tables 3 and 4) micro-channels with roughness may be a result of heterogeneity of the actual roughness, where height of some roughness peaks significantly exceeds its mean value. For example, under conditions of experiment by Pfund et al. [17] the main amplitude of roughness wall was  $\pm 1.9 \mu\text{m}$ , with the maximum peak-value height of approximately  $14.67 \mu\text{m}$ .

The study of forced convection characteristics in rectangular channels with hydraulic diameter of 133–367  $\mu\text{m}$  was performed by Peng and Peterson [22]. In these experiments the liquid velocity ranges from 0.2 to 12 m/s and the Reynolds number was in the range 50–4000. The main results of this study (and subsequent works, for example, Peng and Wang [23]) consisted of the following: (i) drag coefficients for laminar and turbulent flows are inversely proportional  $Re^{1.98}$  and  $Re^{1.72}$ , respectively, (ii) Poiseuille number is not constant; for laminar flow it depends on  $Re$  as follows  $Po \sim Re^{-0.98}$ , (iii) the transi-

tion from laminar to turbulent flow occurs at  $Re$  about 300–700. These results do not agree with those reported by other investigators and are probably incorrect.

Mala and Li [24] investigated experimentally the pressure losses in micro-channels with diameters ranging from 50  $\mu\text{m}$  to 254  $\mu\text{m}$ . The micro-tubes were fabricated from two different materials: silica and stainless steel and had mean surface roughness  $\pm 1.75 \mu\text{m}$ . Thus, the relative roughness changed from 1.36% to 7.0% for the pipes with  $d = 254 \mu\text{m}$  and  $d = 50 \mu\text{m}$ , respectively. The measurements indicate the existence of significant divergence between experimental values of pressure gradient  $\delta P_{exp}$  and values predicted by conventional theory,  $\delta P_{theor}$ . The difference  $\delta P_{exp} - \delta P_{theor}$  depends on the diameter of the micro-tube, as well as on the Reynolds number. At relatively large  $Re$  the dependence  $\delta P(Re)$  is close to linear, whereas at  $Re \sim 10^3$  and  $d < 100 \mu\text{m}$  it is observed significant deviation of  $\delta P(Re)$  from dependence predicted by the Poiseuille equation. It is noteworthy, that there is some difference between pressure losses corresponding to flow in silica and stainless steel micro-tubes: the pressure gradient in silica micro-tube is slightly higher than that for stainless steel. Under conditions corresponding to flow in micro-tubes with the same roughness, such difference points to an existence of some non-hydrodynamic interaction fluid /micro-tube wall. Based on the dependence  $C^* = f(Re)$  it is possible to select two branches corresponding to relatively low

Table 4  
Experimental results of single-phase flow in rough micro-channels

Author	Micro-channel		Fluid	$Re$	Poiseuille number			$Re_{cr}$
	Shape	$d_h$ ( $\mu\text{m}$ )			$Po_{exp}/Po_{th}$	Dependence on $Re$	Relative roughness % ( $k_s/r$ )	
Peng and Peterson [22]	Rectangular	133–367	Water	$2 \times 10^2$ – $4 \times 10^3$	Inversely proportional to $Re^{0.98}$	Inversely proportional to $Re^{0.98}$		300–700
Peng and Wang [23]	Rectangular	133–343	Water	$2 \times 10^2$ – $4 \times 10^3$	Inversely proportional to $Re^{0.98}$	Inversely proportional to $Re^{0.98}$		300–700
Mala and Li [24]	Circular tube	50–254	Deionized water	$10^2$ – $2.1 \times 10^3$	1 for $Re < 500$	Independent on $Re$ at $Re < 500$	1.36–7	$Re_{begin} \sim 300$ –900 $Re_{full} \sim 1000$ –1500
Qu et al. [25]	Trapezoidal	51–169	Deionized ultra filtered water	$10^2$ – $1.6 \times 10^3$	1.15 for $d \sim 60$ 1.3 for $d \sim 120$	Weakly dependent on $Re$	3.6–5.7	$10^3$
Pfund et al. [17]	Rectangular	Depth 257	Deionized water	60–3450	1.25	Independent on $Re$ at $Re < 1700$	Mean 0.74 max 5.7	1700 for 263 $\mu\text{m}$ 2000
Li et al. [15]	Circular tube	128.76–179.8	Deionized water	350–2300	1.15–1.37	Weak depend on $Re$ at $Re < 1500$ $d = 128.7$ $d = 136.5$	3–4	1700
Kandlikar et al. [26]	Circular tube	620–1067	Water	500–2600	$\sim 1$	Independent on $Re$ at $Re < 2 \times 10^3$	0.32–0.71	$\sim 2300$



( $Re < 10^3$ ) and high ( $Re > 10^3$ ) Reynolds numbers. Within the first of these branches the ratio is about of  $C^* = 1.12$ . At  $Re > 10^3$  the ratio  $C^*$  increases with  $Re$ . The latter means that within the ranges of Reynolds numbers  $0 < Re < 10^3$  and  $10^3 < Re < 2 \times 10^3$  the drag coefficient is inversely proportional to  $Re$  and  $(Re)^n$  ( $n < 1$ ), respectively. Thus, the present results correspond (qualitatively) to known data on drag of macro-tube at laminar and developed turbulent flows. The difference between the results corresponding to micro- and macro-roughness tubes manifests itself as displacement to low Reynolds numbers regions where  $\lambda \sim 1/Re$  and  $\lambda = \text{const}$ . We suggest possible reasons of effects mentioned above. The first is an earlier transition from laminar to turbulent flow and the second is the direct effect of roughness on the momentum changes in the liquid layer adjacent to the solid wall.

The hypothesis on earlier transition from laminar to turbulent flow in micro-tubes is based on analysis of the dependence of pressure gradient on Reynolds number. As shown by the experimental data [24], this dependence may be approximated by three power functions: (i)  $\delta P \sim Re^{1.072}$  ( $Re < 600$ ),  $\delta P \sim Re^{1.3204}$  ( $600 < Re < 1500$ ) and  $\delta P \sim Re^{2.0167}$  ( $1500 < Re < 2200$ ) corresponding to laminar, transition and developed turbulent flow, respectively. Naturally, such a phenomenological analysis does not reveal the actual reasons of the displacement of boundary transition to low Reynolds numbers region.

One of the possible ways to account for the influence of the roughness on the pressure drop in micro-tube is to apply a modified-viscosity model to calculate the velocity distribution. Qu et al. [25] performed an experimental study of the pressure drop in trapezoidal silicon micro-channels with relative roughness and hydraulic diameter, ranging from 3.5% to 5.7% and 51  $\mu\text{m}$  to 169  $\mu\text{m}$ , respectively. These experiments showed significant difference between experimental and theoretical pressure gradient.

It was found that pressure gradient and flow friction in micro-channels were higher than that predicted by the conventional laminar flow theory. In a low  $Re$  range, the measured pressure gradient increased linearly with  $Re$ . When  $Re > 500$ , the slope of  $P_x-Re$  relationship increases with  $Re$ . The ratio of  $C^*$  was about 1.3 for micro-channels of hydraulic diameter 51.3–64.9  $\mu\text{m}$  and 1.15–1.18 for micro-channels of hydraulic diameter 114.5–168.9  $\mu\text{m}$ . It was also found that the ratio of  $C^*$  depends on the Reynolds number.

A roughness-viscosity model was proposed in this paper to interpret the experimental data. An effective viscosity  $\mu_{\text{ef}}$  was introduced for this aim as the sum of physical  $\mu$  and imaginary  $\mu_M = \mu_M(r)$ -viscosities. The momentum equation is

$$\frac{1}{r} \frac{\partial}{\partial r} \left( \mu_{\text{ef}} r \frac{\partial u}{\partial r} \right) = \frac{dP}{dx} \quad (7)$$

with no slip conditions on the walls.

In Eq. (7)  $\mu_{\text{ef}} = \mu + \mu_M$  is the effective viscosity,  $\mu_M$  is determined by the semi-empirical correlation  $\mu_M = \mu f(r/k_s, Re, Re_k)$ , ( $Re_k = u_k k_s / \nu$ ,  $Re = u_{\text{av}} d / \nu$ ,  $u_k$  is the velocity at the top of the roughness element,  $u_{\text{av}}$  is the average velocity,  $d$  is micro-tube diameter).

The correlation for  $\mu_M$  contains also an empirical constant that has to be determined by using the experimental data.

Friction factors and friction constant for a 257  $\mu\text{m}$  deep channel with rough bottom plate were measured by Pfund et al. [17]. The width of the channel cut into each spacer was fixed at 1 cm. The micro-channel was 10 cm long, so that the flow profile would be fully developed within at least the middle third of the channel length. The wide channel gave an approximately 2-D flow, thereby simplifying the theoretical description of the flow. Water entered the system through a pressured pulsation damper. The mean amplitude of roughness on the rough polyimide bottom was of  $\pm 1.90 \mu\text{m}$  with a maximum peak-valley height of approximately 14.67  $\mu\text{m}$ . The value of  $C^*$  reached, at low  $Re$ , approximately 1.25. These results were compared to those obtained in the smooth micro-channels with depths ranging from 128 to 521  $\mu\text{m}$ . The authors conclude that the possible treatments of reduction in channel depth at constant roughness, and increase in roughness at constant depth, produced no significant changes in the friction constant. Friction factors in laminar flow were proportional to  $1/Re$  to the degree of accuracy of the experiments. High relative roughness may cause the Poiseuille number to vary with  $Re$ . Uncertainties in the measured constant obscure this effect.

Li et al. [15] studied the flow in a stainless steel micro-tube with diameter 128.76–179.8  $\mu\text{m}$  and relative roughness about 3%–4%. The Poiseuille number for tubes with diameter 128.76  $\mu\text{m}$  and 171.8  $\mu\text{m}$  exceeded the value of  $Po$  corresponding to conventional theory by 37% and 15%, respectively. The critical value of the Reynolds number is close to 2000 for 136.5 and 179.8  $\mu\text{m}$  micro-tubes and about 1700 for micro-tube with diameter 128.76  $\mu\text{m}$ .

The effect of roughness on pressure drop in micro-tubes with diameter 620  $\mu\text{m}$  and 1067  $\mu\text{m}$  and relative roughness 0.71%, 0.58% and 0.321% was investigated by Kandlikar et al. [26]. For the 1067  $\mu\text{m}$  diameter tube, the effect of roughness on pressure drop was insignificant. For the 620  $\mu\text{m}$  tube the pressure drop results showed dependence on the surface roughness.

## 5. Gas flow

Gas flows are flows of compressible fluid that flows under conditions of gas expansion and changing its density, pressure, velocity and temperature along the channel

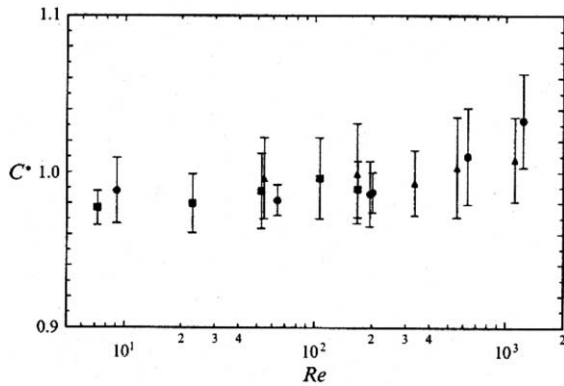


Fig. 5. Normalized Poiseuille number as a function of the Reynolds number in 11.04  $\mu\text{m}$  deep channel. (●) nitrogen, (■) helium, (▲) argon [27].

length [37]. There is paucity of theoretical study of laminar gas flows in micro-channels. Berg et al. [38] considered this problem in conjunction with calculation of the viscosity from the data on mass flow rate. An experimental and theoretical investigation of low Reynolds number, high subsonic Mach number, compressible gas flow in channels was presented by Harley et al. [27]. Nitrogen, helium, and argon gases were used. By means of analytical and numerical solutions of the problem the detailed data on velocity, density and temperature distributions along micro-channels axis were obtained. The effect of the Mach number on profiles of axial and transversal velocities and temperature was revealed. The friction factor in trapezoidal micro-channels typically 100  $\mu\text{m}$  wide,  $10^4$   $\mu\text{m}$  long and depth ranging from 0.5  $\mu\text{m}$  to 20  $\mu\text{m}$ , was measured. The measurements of the pressure drop for the flow of nitrogen, helium and argon were carried

out by varying the Knudsen number from  $10^{-3}$  to 0.4, the inlet and exit Mach numbers 0.07 and 0.15, and 0.22, and 0.84, respectively and  $Re < 2 \times 10^3$ . Fig. 5 shows that under conditions which correspond to continuum flow, the measured value of friction factor is close to that which is predicted by conventional theory: average friction constant was within 3% of the theoretical value for fully developed incompressible flow.

## 6. Transition from laminar to turbulent flow

The data on critical Reynolds numbers in micro-channels of circular and rectangular cross section are presented in Tables 5 and 6, respectively. We also point out geometrical characteristics of the micro-channels and the methods used for determination of the critical Reynolds number.

For the most part of the experiments one can conclude that transition from laminar to turbulent flow in smooth and rough circular micro-tubes occurred at Reynolds numbers about  $Re_{cr} = 2000$  corresponding to those in macro-channels. Note, that there were reported also other results. According to Yang et al. [16]  $Re_{cr}$  derived from dependence of pressure drop on Reynolds number varied from  $Re_{cr} = 1200$  to  $Re_{cr} = 3800$ . The low value was obtained for the flow in a tube of diameter 4.01 mm, whereas the high one was obtained for flow in a tube of diameter 0.502 mm. These results look highly questionable since they contradict the data related to the flow in tubes of diameter  $d > 1$  mm. Really, the tube of diameter 4.01 mm may be considered to a macro-tube, in which the critical Reynolds number is about 2,000. A possible reason of this inconsistency arises from the method of determination of  $Re_{cr}$ . The data discussed

Table 5  
Critical Reynolds number in circular micro-channels

Author	Micro-channel				$Re_{cr}$	Remarks: considered characteristic
	Smooth/roughness	$d$ ( $\mu\text{m}$ )	$\ell$ (mm)	$\frac{\ell}{d}$		
Li et al. [15]	Smooth	79.9–449	14.56–118.9	182–714	2000	Friction factor
Yang et al. [16]	Smooth	502–4010	200–1000	244–567	2200	Friction factor ( $Re_{cr} \sim 1200$ –3800 pressure gradient)
Maynes and Webb [20]	Smooth	705	141.9	201.2	2200	Friction factor
Mala and Li [24]	Smooth	50–254	55–88		300–900 Fully developed turbulent flow at $Re > 1000$ –1500	Pressure gradient
Li et al. [15]	Rough	128–179.8	39.30–84.26	305–470	1700–1900	Friction factor
Kandlikar et al. [26]	Rough	620–1032	–	–	2000	Friction factor
Sharp and Adrian [14]	Smooth	50–247	–	–	1800–2200	Friction factor, centerline velocity, rms of centerline velocity fluctuations

Table 6  
Critical Reynolds number in rectangular micro-channels

Author	Micro-channel				$Re_{cr}$	Remarks: considered characteristic
	Smooth/roughness	$H$ ( $\mu\text{m}$ )	$\ell$ (mm)	$\frac{\ell}{H}$		
Xu et al. [18]	Smooth	15.4	10–50	367–1700	1500	Friction factor
Pfund et al. [17]	Smooth	128–1050	100	95–781	1700 for $H = 263$ $\mu\text{m}$ 2000–2100 for $H = 521$ $\mu\text{m}$	Friction factor
Pfund et al. [17]	Rough	257	100	389	1700	Friction factor
Qu [25]	Rough (trapezoidal)	28–113.84	30	263–1071	–	Pressure gradient
Peng and Peterson [22]	Rough	133–367			200–700	Friction factor

above were obtained by analysis of the dependences of the pressure gradient on the Reynolds number. The same experimental data presented by Yang et al. [16] in the form of dependence of the friction factor on the Reynolds number, clearly showed that  $Re_{cr}$  is about 2000. Mala and Li [24] using the method based on dependence of the pressure drop gradient on Reynolds number obtained  $Re_{cr} = 300$ –900.

The transition from laminar to turbulent flow in micro-channels with diameter ranging from 50  $\mu\text{m}$  to 247  $\mu\text{m}$  was studied by Sharp and Adrian [14]. The transition to turbulent flow was studied for liquids of different polarities in glass micro-tubes having diameters between 50 and 247  $\mu\text{m}$ . The onset of transition occurred at Reynolds number about 1800–2000, as indicated by greater-than-laminar pressure drop and micro-PIV measurements of mean velocity and rms velocity fluctuations at the center line.

In the laminar region the rms of streamwise velocity fluctuation was expected to be zero. Fig. 6 shows that the first evidence of transition, in the form of an abrupt increase in the rms, occurs between  $1800 \leq Re \leq 2200$ ,

in full agreement with the flow resistance data. There was no evidence of transition below these values. Thus, the behavior of the flow in micro-tubes, at least down to 50  $\mu\text{m}$  diameter, shows no perceptible differences with macro-scale flow.

## 7. Effect of measurement accuracy

In experiments of flow and heat transfer in micro-channels, some parameters, such as the flow rate and channel dimensions are difficult to measure accurately because they are very small. For single-phase flow in micro-channels the uncertainty of  $\lambda Re$  is [39,41]

$$\frac{\delta(\lambda Re)}{\lambda Re} = \left\{ \left( \frac{\delta(\Delta P)}{\Delta P} \right)^2 + \left( \frac{4(\delta d_h)}{d_h} \right)^2 + \left( \frac{\delta(\ell)}{\ell} \right)^2 + \left( \frac{\delta(m)}{m} \right)^2 \right\}^{1/2} \quad (8)$$

where  $m$ ,  $\Delta P$ ,  $d_h$ ,  $\ell$  are the mass flow rate, pressure drop, hydraulic diameter and channel length. Eq. (8) shows that the channel hydraulic diameter measurement error may play very important part in the uncertainty of the product  $\lambda Re$ . For example, in experiments of measuring the friction factor in a circular glass micro-tube by Guo and Li [39], they initially measured the diameter as 84.7  $\mu\text{m}$  using a 40 $\times$  microscope. The data reduction with this measured diameter showed that the friction factors were larger than that predicted by conventional theory. The averaged value of the diameter measured using a 400 $\times$  microscope and scanning electronic microscope for the same micro-tube was only 80.0  $\mu\text{m}$ . With this more accurate value of the diameter, the friction factors obtained from the experimental data were in good agreement with the conventional values.

## 8. Discussion

### 8.1. General remarks

The data presented in the previous chapters, as well as the data of investigations of single-phase forced

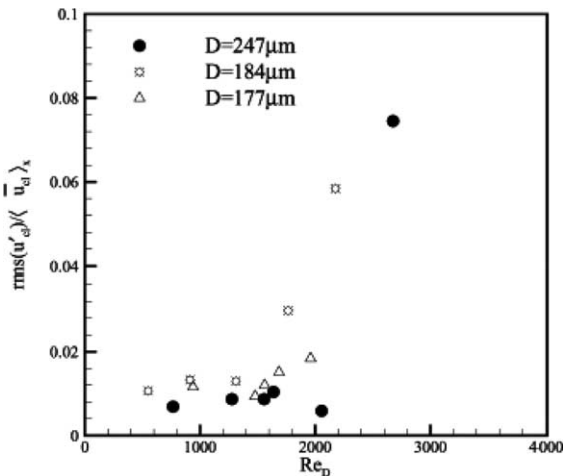


Fig. 6. Measured rms of the center line velocity, divided by measured velocity versus the Reynolds number [14].

convection heat transfer in micro-channels (e.g. [40–42]) show there exists a number of principal problems related to micro-channel flows. Among them there are: (i) dependence of pressure drop on Reynolds number, (ii) value of the Poiseuille number and its consistency with prediction of conventional theory, (iii) value of the critical Reynolds number and its dependence on roughness, fluid properties etc.

All available experimental data (except data by Peng and Peterson [22], Peng and Wang [23]) show that the drag coefficient is inversely proportional to the Reynolds number i.e.  $\lambda = \frac{\text{const}}{Re}$ . The value of the constant depends on the micro-channel shape only and agrees fairly well with the result of a dimensional analysis carried out by Sedov [43] and analytical solution of the problem [29]. The qualitative difference between the data by Peng and Peterson [22] and Peng and Wang [23] from the data of the other known experimental and theoretical researches is, probably, due to experimental uncertainties.

Several groups of experiments related to transition from laminar to turbulent flow can be selected (Tables 5 and 6):

Li et al. [15], Yang et al. [16], Maynes and Webb [20], Kandlikar et al. [26] and Sharp and Adrian [14] obtained the critical Reynolds number in the range  $Re_{cr} \approx 1700$ –2300.

Li et al. [15], Xu et al. [18], Pfund et al. [17] also obtained the critical Reynolds number  $Re_{cr} \approx 1500$ –1900.

Peng and Peterson [22], Peng and Wang [23], Mala and Li [24] obtained the anomalous low critical Reynolds number  $Re_{cr} \approx 200$ –900.

The critical Reynolds number  $Re_{cr} = 1700$ –2300 agree fairly well with the well known values of  $Re_{cr}$  corresponding to flow in macro-channels [46–48,1]. This result is not unexpected since from a simple physical consideration it is obvious that under conditions of continuous model with viscous fluid/wall interaction and moderate flow velocity, when energy dissipation is negligible, there are no reasons for a decrease in the critical Reynolds number. Taking into consideration dimension of the characteristic parameters, we obtain in this case that the life time of vortices is inversely proportional to the fluid viscosity

$$t \sim \frac{d^2}{\nu} \quad (9)$$

The characteristic hydrodynamic time,  $t_h$  is

$$t_h = \frac{\ell}{u} \quad (10)$$

and

$$\frac{t}{t_h} \sim \frac{Re}{\ell} \quad (11)$$

where  $\bar{\ell} = \ell/d$ .

The ratio of  $t/t_h$ , which is characteristic of possibility of existence of vortices, does not depend on the micro-channel diameter and is fully determined by the Reynolds number and  $\ell/d$ . The lower value of  $Re$  at which  $\frac{t}{t_h} \geq 1$  can be treated as a threshold. As was shown by Darbyshire and Mulling [49], under conditions of artificial disturbance of pipe flow, a transition from laminar to turbulent flow is not possible for  $Re < 1700$  even with very large amplitude of disturbances.

A direct study of transition from laminar to turbulent flow in micro-tubes was performed by Sharp and Adrian [14]. The measurements of mean velocity and rms of velocity fluctuations showed that the value of Reynolds number, at which the transition from laminar to turbulent flow occurs, is about  $Re_{cr} \approx 1800$ –2200. Thus, the measurements of integral flow characteristics, as well as mean velocity and rms of velocity fluctuations testify that the critical Reynolds number is the same as  $Re_{cr}$  in macroscopic Poiseuille flow. Some decrease of critical Reynolds number up to  $Re \sim 1500$ –1700, in the second group of research, may be due to energy dissipation. The energy dissipation leads to an increase in fluid temperature. In both cases it leads to change of fluid viscosity: increase in gas viscosity, and decrease in liquid viscosity. Accordingly, the Reynolds number based on the inlet flow viscosity differs from that based on local viscosity at a given point in the micro-channel.

There is a significant scatter between the values of the Poiseuille number in micro-channel flows fluids with different physical properties. The results presented in Table 1 for deionized water flow, in smooth micro-channels, are very close to values predicted by conventional theory. Significant discrepancy between theory and experiment was observed in the cases when fluid with unknown physical properties was used (tap water, etc.). If the liquid contains even very small amounts of ions, the electrostatic charges on the solid surface will attract the counter ions in the liquid to establish an electrical field. Fluid–surface interaction can be put forward as an explanation of the Poiseuille number increase by the fluid ionic coupling with the surface [44,45,34].

The results obtained by Brutin and Tadrif [44] showed a clear effect of the fluid on the Poiseuille number. Fig. 7 shows results of experiments that were done in the same experimental setup for hydraulic diameters of 152 and 262  $\mu\text{m}$ , using distilled water and tap water. The ion interactions with surface can be proposed to explain such differences. Tap water contains more ions such as  $\text{Ca}^{2+}$ ,  $\text{Mg}^{2+}$ , which are 100–1000 times more concentrated than  $\text{H}_3\text{O}^+$  or  $\text{OH}^-$ . In distilled water only  $\text{H}_3\text{O}^+$  and  $\text{OH}^-$  exist in an equal low concentrations. The anions and cations interactions with the polarized surface could modify the friction factor. It is valid only in case of a non-conducting surface.

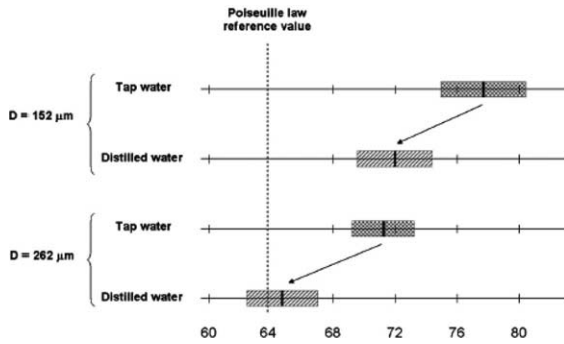


Fig. 7. Poiseuille number for distilled and tap water using 152 and 256  $\mu\text{m}$  diameter micro-tubes with a classical fused silica inner surface [44].

## 8.2. Thermal effects

Gruntfest et al. [50] showed that the thermal effects due to energy dissipation of liquid flow in a pipe lead to significant transformation of the flow field. It occurs due to the dependence of liquid density on temperature, distortion of velocity profile and development of flow instability and its transition to turbulence. The influence of energy dissipation on thermohydrodynamic characteristics of the liquid flow in micro-channels was considered by Tso and Mahulikar [51–53]. The detailed analysis of viscous dissipation effects in micro-tubes and micro-channels was performed by Koo and Kleinstreuer [54]. It was shown that viscous dissipation becomes significant for fluids with low specific capacities and high viscosities, even in relatively low Reynolds number flows. The channel size, aspect ratio of the channel to its lengths, as well as the Reynolds and Brinkman numbers are important factors that determine the effect of viscous dissipation.

The behavior of liquid flow in micro-tubes and channels depends not only on the absolute value of the viscosity but also on its dependence on temperature. The non-linear character of this dependence is a source of an important phenomenon—hydrodynamic thermal explosion (THE): sharp change of flow parameters at small temperature disturbances due to viscous dissipation. This is accompanied by radical changes of flow characteristics. Bastanjian et al. [55] showed that under certain conditions the steady-state flow cannot exist, and an oscillatory regime begins.

We estimated the effect of energy dissipation on liquid heating and values of flow parameters corresponding to arising oscillations in the flow. We assume that the density of the fluid and its thermal conductivity are constant. Then the energy equation takes the form

$$\rho(v\nabla)h = k\nabla^2 T + \Phi \quad (12)$$

where  $h$  is the enthalpy,  $k$  is the thermal conductivity,  $\Phi$  is the energy dissipation. The present analysis is restricted to estimation of the maximum heating of the liquid that corresponds to adiabatic flow in a micro-channel.

Assume that the flow in a long micro-channel is fully developed and convective heat transfer along micro-channel axis exceeds the conductive one. Taking into account that

$$u = u_m(1 - \eta^2) \quad (13)$$

and

$$u_m = 2\bar{u}, \quad u_m = \frac{3}{2}\bar{u} \quad (14)$$

for axisymmetric and plane flows, respectively, we obtain from Eq. (12) the following estimation for adiabatic rise of the liquid temperature

$$\frac{\Delta T}{T_{in}} = 2 \frac{v^2}{r_0^2} \left( \frac{\ell}{r_0} \right) \frac{Re}{c_p T_{in}} \quad (15)$$

for circular micro-channel, and

$$\frac{\Delta T}{T_{in}} = 3 \frac{v^2}{h_0 d_h} \frac{\ell}{h_0} \frac{Re}{c_p T_{in}} \quad (16)$$

for plane micro-channel

where  $u$  and  $u_m$  are the local and maximum velocity,  $\bar{u} = z \frac{1}{r_0} \int_0^{r_0} u r dr$  and  $\bar{u} = \frac{1}{h_0} \int_0^{h_0} u dy$  are the average velocity for axisymmetric and plane flows,  $\eta = r/r_0$  and  $\eta = y/h_0$ ,  $r_0$  is the micro-tube radius,  $h_0 = H/2$ ,  $H$  is the depth of the plane micro-channel,  $\nu$  is the kinematic viscosity,  $c_p$  is the specific heat of liquid,  $T_{in}$  is the inlet temperature,  $\Delta T = T_{out} - T_{in}$  is the difference between outlet and inlet temperatures. The difference between outlet and inlet temperatures essentially depends on the heat capacity of the fluid and sharply increases as  $c_p$  decreases.

The estimation of adiabatic increase in the liquid temperature in circular micro-tubes with diameter ranging from 15  $\mu\text{m}$  to 150  $\mu\text{m}$ , under experimental conditions by Judy et al. [21], are presented in Table 7. The calculations were carried out for water, isopropanol and methanol flows, respectively, at initial temperature  $T_{in} = 298 \text{ K}$  and  $\nu = 8.7 \times 10^{-7} \text{ m}^2/\text{s}$ ,  $2.5 \times 10^{-6} \text{ m}^2/\text{s}$ ,  $1.63 \times 10^{-6} \text{ m}^2/\text{s}$ ,  $c_p = 4178 \text{ J/kg K}$ ,  $2606 \text{ J/kg K}$ ,  $2531 \text{ J/kg K}$ . The lower and higher values of  $\Delta T/T_{in}$  correspond to limiting values of micro-channel length and Reynolds numbers.

Table 7 shows that adiabatic heating of liquid in micro-tubes can reach tens of degrees: mean temperature rising  $\langle \Delta T \rangle = \langle T \rangle - T_{in}$  ( $\langle T \rangle = (T_{out} + T_{in})/2$ ) is about 9  $^\circ\text{C}$ , 122  $^\circ\text{C}$  and 37  $^\circ\text{C}$  for the water ( $d = 20 \mu\text{m}$ ), isopropanol ( $d = 20 \mu\text{m}$ ) and methanol ( $d = 30 \mu\text{m}$ ) flows.

Energy dissipation affects significantly also the temperature of gas flow. In order to estimate adiabatic

Table 7  
Adiabatic increase in the liquid temperature in a circular micro-tube: data from [21]

$d$ ( $\mu\text{m}$ )	$\ell$ (m)	$Re$	$\frac{\Delta T}{T_{in}} \times 10^2$		
			Water	Isopropanol	Methanol
15	0.036	34–41	0.353–0.425	4.67–5.63	2.04–2.46
20	0.03–0.05	18–989	0.0657–6.01	0.87–79.6	0.38–34.8
30	0.04–0.07	8–1716	0.0115–4.33	0.153–57.3	0.067–25.1
40	0.05–0.37	17–769	0.0129–4.32	0.171–57.2	0.075–25.1
50	0.07–0.29	44–1451	0.0239–3.27	0.317–43.3	0.139–18.9
75	0.3–0.39	146–1883	0.1–1.69	1.34–22.4	0.585–9.81
100	0.39	109–1858	0.0413–0.70	0.55–9.3	0.239–4.08
150	0.2–0.3	137–1540	0.0079–0.133	0.105–1.8	0.046–0.77

Table 8  
Characteristics of a micro-channel [27]

Top width ( $\mu\text{m}$ )	Bottom width ( $\mu\text{m}$ )	Depth (mm)	Hydraulic diameter $d_h$ ( $\mu\text{m}$ )	Length (mm)	$\frac{\ell}{d_h}$	Gases tested
94.4	80.0	11.04	19.18	10.18	992.1	$\text{N}_2$ , He, Ar

increase in gas temperature in micro-channels we used the approximate expression for  $\Delta T/T_{in}$  which follows from Eq. (16) with the assumption that  $k\nabla^2 T \sim 0$ ,  $h = c_p T$ ,  $\Phi = \mu \left(\frac{du}{dy}\right)^2 \sim \mu \frac{u^2}{\delta^2}$  and  $\frac{dT}{dx} = \frac{T_{out}-T_{in}}{\ell}$

$$\frac{\Delta T}{T_{in}} = \frac{v^2}{2\delta^2 c_p T_{in}} \left(\frac{\ell}{\delta}\right) Re \tag{17}$$

where  $\delta = d_h/2$ ,  $u = \bar{u}$  are characteristic size and velocity,  $Re = \bar{u}2\delta/v$ .

For parameters presented in Table 8, that correspond to conditions in the experiments by Harley et al. [27], mean increase of temperature in nitrogen, helium and argon ( $Re_{\text{Nr}} = 60$ ,  $Re_{\text{Hel}} = 20$ ,  $Re_{\text{Ar}} = 50$ ,  $M \sim 0.1$ ) is about 50 °C–200 °C (Table 9).

Energy dissipation leads to significant increase in gas viscosity and decrease in the actual Reynolds number  $Re_{ac}$  defined by the mean the viscosity  $v_{ac} = (v_{out} + v_{in})/2$  compared to that defined by inlet parameters.

Under conditions of real experiments, the thermal regime of the flow determines not only the energy dissipation, but also the heat losses to the micro-channel wall. In this case increase in the fluid temperature depends on the relation between intensity of heat released by energy dissipation and heat losses due to heat transfer [54]. This does not distort the qualitative picture of the phenomenon—in all cases energy dissipation leads to temperature

growth, changing viscosity of fluid and actual Reynolds number. Moreover, in certain cases, energy dissipation leads to radical transformation of flow and transition from stable to oscillatory regime.

### 8.3. Oscillatory regimes

To estimate the parameters resulting in such transition we use the approach by Bastanjian et al. [55] and Zel'dovich et al. [56]. The momentum and energy equation for steady and fully developed flow in circular tube are

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \mu \frac{\partial u}{\partial r} \right) - \frac{\partial P}{\partial x} = 0 \tag{18}$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{2} \frac{\partial T}{\partial r} + \frac{\mu}{kT} \left( \frac{\partial u}{\partial r} \right)^2 = 0 \tag{19}$$

where  $\mu = \mu(T)$ .

The dependence of the liquid viscosity on the temperature is given by Frenkel [57]

$$\mu = \mu_0 \exp\left(\frac{E}{RT}\right) \tag{20}$$

Using the Frank–Kamenetskii [58] transformation we present the relation in (20) in the following form [56]

Table 9  
Increase in adiabatic temperature in gas flows

Gases	$\frac{\Delta T}{T_{in}}$	$\frac{(\Delta T)}{T_{in}}$	$\frac{v_{ac}}{v_{in}}$	$\frac{Re_{ac}}{Re_{in}}$	$\langle T \rangle$ , °C ( $T_{in} = 300$ K)
Nitrogen	0.37	0.18	~1.35	~0.74	355
Helium	1.522	0.761	~2.87	~0.35	528
Argon	0.096	0.048	~1.09	~0.919	314

$$\mu = \mu_0 \exp\left(\frac{E}{RT_0}\right) \exp\left(-\frac{E(T-T_0)}{RT_0^2}\right) \quad (21)$$

Introducing new variables

$$\xi = \left(\frac{r}{r_0}\right)^2, \quad \theta = \frac{E(T-T_0)}{RT_0^2} \quad (22)$$

we reduce the energy equation (19) to the following form (Zel'dovich et al. [56])

$$\frac{\partial^2 \theta}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial \theta}{\partial \xi} + \chi e^\theta = 0 \quad (23)$$

where the dimensionless parameter  $\chi$  is

$$\chi = \frac{(\partial_x P)^2 r_0^4}{16\mu_0 k} \frac{E}{RT_0^2} e^{-\frac{E}{RT_0}} \quad (24)$$

the boundary conditions corresponding to constant temperature are

$$\theta(1) = 0, \quad \left(\frac{\partial \theta}{\partial \xi}\right)_{\xi=0} = 0 \quad (25)$$

A steady state solution of Eq. (23) exists only for  $\chi \leq 2$ . At  $\chi > 2$  hydrodynamic thermal explosion occurs and oscillatory flow takes place.

Bearing in mind that  $\Delta P = \lambda P \frac{\ell}{d} \frac{\rho u^2}{2}$ ,  $\lambda = \frac{64}{Re^4}$ ,  $u = \frac{Re v}{d}$ ,  $\partial_x P = \frac{\Delta P}{\ell} = 32 \frac{\rho v^2}{d^3} Re$ , and assuming that  $\frac{\mu_0}{\rho} \simeq \nu_0 \approx \nu$  we obtain

$$\chi = \frac{Re^2}{r_0^2} \frac{\nu_0^2}{c_p T_0} \frac{E}{RT_0} Pr e^{-\frac{E}{RT_0}} \quad (26)$$

or

$$Re_{cr} = \left(\frac{\chi_{cr}}{N}\right)^{1/2} r_0 \quad (27)$$

where  $\chi_{cr} = 2$ ,  $N = Pr \frac{\nu_0^2}{c_p T_0} \frac{E}{RT_0} e^{-\frac{E}{RT_0}}$ .

The physical properties of water and transformer oil is given in Table 10 [59]. Using the data in Table 10 we estimate the critical Reynolds number for flows of water and transformer oil

$$Re_{cr} = 1.94 \times 10^{10} r_0 \quad (28)$$

for water, and

$$Re_{cr} = 2.5 \times 10^8 r_0 \quad (29)$$

for transformer oil where  $r_0$  [m] is the micro-channel radius.

The critical Reynolds number depends significantly on physical properties of the liquid (kinematic viscosity,

heat capacity and the Prandtl number), and micro-channel radius. For flow of high-viscous liquids (for example, transformer oil) in micro-channels of  $r_0 < 10^{-5}$  m, the critical Reynolds number is less than 2300. Under these conditions, oscillatory regime occurs at all  $Re$  corresponding to laminar flow. The existence of velocity fluctuations does not testify to a change of the flow regime from laminar to turbulent. This phenomenon shows only the occurrence of oscillatory laminar flow. In water flow (small kinematic viscosity, large heat capacity) rising of velocity oscillations is not possible at any realistic  $r_0$ .

## 9. Conclusions

The main aim of the present analysis is to verify the capacity of conventional theory to predict the hydrodynamic characteristics of laminar Newtonian incompressible flows in micro-channels in the range of hydraulic diameter from  $d_h = 15$  to  $d_h = 4010 \mu\text{m}$ , Reynolds number from  $Re = 10^{-3}$  up to  $Re = Re_{cr}$ , and Knudsen number from  $Kn = 0.001$  to  $Kn = 0.4$ . The following conclusions can be drawn from this study:

The comparison of experimental results to those obtained by conventional theory is correct when the experimental conditions were consistent with the theoretical ones. The experimental results corresponding to these requirements agree quite well with the theory.

For single-phase fluid flow in smooth micro-channels of hydraulic diameter from 15 to 4010  $\mu\text{m}$ , in the range of Reynolds number  $Re < Re_{cr}$ , the Poiseuille number,  $Po$  is independent of the Reynolds number,  $Re$ .

For single-phase gas flow in micro-channels of hydraulic diameter from 101 to 4010  $\mu\text{m}$ , in the range of Reynolds number  $Re < Re_{cr}$ , Knudsen number  $0.001 \leq Kn \leq 0.38$ , Mach number  $0.07 \leq Ma \leq 0.84$ , the experimental friction factor agrees quite well with the theoretical one predicted for fully developed laminar flow.

The behavior of the flow in micro-channels, at least down to 50  $\mu\text{m}$  diameter, shows no differences with macro-scale flow. For smooth and rough micro-channels with relative roughness  $0.32\% \leq k_s \leq 7\%$  the transition from laminar to turbulent flow occur between  $1800 \leq Re_{cr} \leq 2200$ , in full agreement with flow visualization and flow resistance data. In the articles used for the present study there was no evidence of transition below these results.

Table 10  
Physical properties of the fluids [59]

Fluid	$\mu \times 10^3$ (Pa s)	$\rho$ (kg/m <sup>3</sup> )	$\nu \times 10^6$ (m <sup>2</sup> /s)	$c_p$ (J/kg K)	$k$ (W/mK)	$Pr$	$E/R$ (K)
Water	1.0	998	1.003	4190	602	6.94	2090
Transformer oil	21.0	879	24.0	1710	111	326	3352

The relation of hydraulic diameter to channel length and the Reynolds number are important factors that determine the effect of the viscous energy dissipation on flow parameters.

Under certain conditions the energy dissipation may lead to an oscillatory regime of laminar flow in micro-channels. The oscillatory flow regime occur in micro-channels at Reynolds numbers less than  $Re_{cr}$ . In this case the existence of velocity fluctuations does not testify to change from laminar to turbulent flow.

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### References

- [1] H. Schlichting, *Boundary Layer Theory*, McGraw Hill Book Company, New York, 1979.
- [2] G. Hagen, *Über die Bewegung des Wassers in engen Zylindrischen Rohren*, *Pogg. Ann.* 46 (1839) 423–442.
- [3] J. Poiseuille, *Recherches expérimentelles tubes de très petits diamètres*, *Comptes Rendus* 11 (1840) 961–967, 1041–1048.
- [4] C.-M. Ho, Y.-C. Tai, *Micro-electro-mechanical systems (MEMS) and fluid flows*, *Ann. Rev. Fluid Mech.* 30 (1998) 579–612.
- [5] M. Gad-el-Hak, *The fluid mechanics of microdevices. The Freeman Scholar Lecture*, *J. Fluid Eng.* 121 (1999) 5–33.
- [6] D.B. Tuckerman, *Heat transfer micro-structure for integrated circuits*, PhD thesis, Department of Electrical Engineering, Stanford University, 1984.
- [7] F.P. Incropera, *Liquid Cooling of Electronic Devices by Single-phase Convection*, John Wiley & Sons Inc., New York, 1999.
- [8] A.B. Duncan, G.P. Peterson, *Review of microscale heat transfer*, *Appl. Mech. Rev.* 47 (1994) 397–428.
- [9] B. Plam, *Heat transfer in microchannels*, in: *Heat Transfer and Transport Phenomena in Microscale*, Banff., October 2000, pp. 54–64.
- [10] H. Herwig, *Flow and heat transfer in micro systems. Everything Different or Just Smaller*, *ZAMM* 0 (2000) 1–10.
- [11] H. Herwig, O. Hausner, *Critical view on new results in micro-fluid mechanics an example*, *Int. J. Heat Mass Transfer* 46 (2003) 935–937.
- [12] M. Gad-el-Hak, *Comments or critical view on new results in micro-fluid mechanics*, *Int. J. Heat Mass Transfer* 46 (2003) 3941–3945.
- [13] K.V. Sharp, R. Adrian, J. Santiago, J.I. Molho, *Liquid flows in micro-channels*, in: *CRC Handbook of MEMS*, CRC Press, Boca Raton, FL, 2001, pp. 6.1–6.38.
- [14] K.V. Sharp, R.J. Adrian, *Transition from laminar to turbulent flow in liquid filled microtubes*, *Exp. Fluids* 36 (2004) 741–747.
- [15] Z.X. Li, D.X. Du, Z.Y. Guo, *Experimental study on flow characteristics of liquid in circular micro-tubes*, *Microscale Thermophys. Eng.* 7 (2003) 253–265.
- [16] C.Y. Yang, J.C. Wu, H.T. Chien, S.R. Lu, *Friction characteristics of water, R-134a, and air in small tubes*, *Microscale Thermophys. Eng.* 7 (2003) 335–348.
- [17] D. Pfund, D. Rector, A. Shekarriz, *Pressure drop measurements in a micro-channel*, *AIChE J.* 46 (2000) 1496–1507.
- [18] B. Xu, K.T. Ooi, N.T. Wong, W.K. Choi, *Experimental investigation of flow friction for liquid flow in micro-channels*, *Int. Comm. Heat Transfer* 27 (8) (2000) 1165–1176.
- [19] H.Y. Wu, P. Cheng, *Friction factors in smooth trapezoidal silicon micro-channels with different aspect ratios*, in: *J. Heat Mass Transfer* 46 (2003) 2519–2525.
- [20] D. Maynes, A.R. Webb, *Velocity profile characterization in sub-diameter tubes using molecular tagging velocimetry*, *Exp. Fluids* 32 (2002) 3–15.
- [21] J. Judy, D. Maynes, B.W. Webb, *Characterization of frictional pressure drop for liquid flows through micro-channels*, *Int. J. Heat Mass Transfer* 45 (2002) 3477–3489.
- [22] X.F. Peng, G.P. Peterson, *Convective heat transfer and friction for water flow in micro-channel structures*, *Int. J. Heat Mass Transfer* 39 (1996) 2599–2608.
- [23] X.F. Peng, B.X. Wang, *Forced convection and boiling characteristics in micro-channels*, in: *Heat Transfer 1998 Proceedings of 11th IHTC*, vol. 11, Kyongju, Korea, 23–28 August 1998, pp. 371–390.
- [24] Gh.M. Mala, D. Li, *Flow characteristics of water in micro-tubes*, *Int. J. Heat Fluid Flow* 20 (1999) 142–148.
- [25] W. Qu, Gh.M. Mala, D. Li, *Pressure driven water flows in trapezoidal silicon micro-channels*, *Int. J. Heat Mass Transfer* 43 (2000) 353–364.
- [26] S.G. Kandlikar, S. Joshi, S. Tian, *Effect of surface roughness on heat transfer and fluid flow characteristics at low Reynolds numbers in small diameter tubes*, *Heat Transfer Eng.* 24 (2003) 4–16.
- [27] J.C. Harley, Y. Huang, H.H. Bau, J.N. Zewel, *Gas flow in micro-channels*, *J. Fluid Mech.* 284 (1995) 257–274.
- [28] S.S. Hsieh, H.H. Tsai, C.Y. Lin, C.F. Huang, C.M. Chien, *Gas flow in long micro-channel*, *Int. J. Heat Mass Transfer* 47 (2004) 3877–3887.
- [29] L.G. Loitsianskii, *Mekhanika Zhidkosti i Gasov*, Nauka, Moscow, 1973.
- [30] R.K. Shah, A.L. London, *Laminar Flow Forced Convection in Duct*, Academic, 1978.
- [31] F.M. White, *Fluid Mechanics*, third ed., McGraw Hill, New York, 1994.
- [32] H.B. Ma, G.P. Peterson, *Laminar friction factor in microscale ducts of irregular cross section*, *Microscale Thermophys. Eng.* 1 (1997) 253–265.
- [33] D. Lelea, S. Nishio, K. Takano, *The experimental research on micro-tube heat transfer and fluid flow of distilled water*, *Int. J. Heat Mass Transfer* 47 (2004) 2817–2830.



- [34] I. Papautsky, J. Brazzle, T. Ameel, B. Frazier, Laminar fluid behavior in micro-channels using micro-polar fluid theory, *Sensors Actuators* 73 (1999) 101–108.
- [35] H.H. Cui, Z.H. Silber-Li, S.N. Zhu, Flow characteristics of liquids in micro-tubes driven by high pressure, *Phys. Fluids* 16 (2004) 1803–1810.
- [36] I. Tani, Boundary layer transition, *Ann. Rev. Fluid Mech.* 1 (1969) 169–196.
- [37] A.K. Shapiro, *The Dynamics and Thermodynamics of Compressible Fluid Flow*, Vols. 1 and 2, John Wiley, 1953.
- [38] H.R. van den Berg, C.A. ten Seldam, P.S. van der Gulik, Compressible laminar flow in a capillary, *J. Fluid Mech.* 246 (1993) 1–20.
- [39] Z.Y. Guo, Z.X. Li, Size effect on micro-scale single phase flow and heat transfer, in: *Proceedings of the Twelfth International Heat Transfer Conference*, Grenoble, France, 18–23 August 2002.
- [40] D.K. Bailey, T.A. Ameel, R.O. Warrington Jr., T.I. Savoie, Single phase forced convection heat transfer in micro-geometries, IECEC Paper No. ES-396, ASME, 1995, pp. 301–310.
- [41] Z.Y. Guo, Z.X. Li, Size effect on micro-scale single-phase flow heat transfer, *Int. J. Heat Mass Transfer* 46 (2003) 149–159.
- [42] G.P. Celata, M. Gumo, G. Zummo, Thermal-hydraulic characteristics of single-phase flow in capillary pipes, *Exp. Thermal Fluid Sci.* 28 (2004) 87–95.
- [43] L.I. Sedov, *Similarity and Dimensional Methods in Mechanics*, 10th ed., CRC Press, Boca Raton, FL, 1993.
- [44] D. Brutin, L. Tadrist, Experimental friction factor of a liquid flow in micro-tubes, *Phys. Fluids* 15 (2003) 653–661.
- [45] L. Ren, W. Qu, D. Li, Interfacial electrokinetic effects on liquid flow in micro-channels, *Int. J. Heat Mass Transfer* 44 (2001) 3125–3134.
- [46] E.R. Lindgren, The transition process and other phenomena in viscous flow, *Arkiv fur Physik* 12 (1958) 1–169.
- [47] R.J. Leite, An experimental investigation of the stability of Poiseuille flow, *J. Fluid Mech.* 5 (1959) 81–96.
- [48] I.J. Wygnanski, F.H. Champagne, On transition in a pipe. Part 1. The origin of puffs and slugs and the flow in a turbulent slug, *J. Fluid Mech.* 59 (1973) 281–351.
- [49] A.G. Darbyshire, T. Mullin, Transition to turbulence in constant-mass-flux pipe flow, *J. Fluid Mech.* 189 (1995) 83–114.
- [50] I.J. Gruntfest, J.P. Young, N.L. Jonson, Temperatures generated by the flow of liquids in pipes, *J. Appl. Phys.* 35 (1964) 18–23.
- [51] C.P. Tso, S.P. Mahulikar, The use of the Brinkman number for single phase forced convective heat transfer in micro-channels, *Int. J. Heat Mass Transfer* 41 (1998) 1759–1769.
- [52] C.P. Tso, S.P. Mahulikar, The role of the Brinkman number in analysis flow transition in micro-channel, *Int. J. Heat Mass Transfer* 42 (1999) 1813–1833.
- [53] C.P. Tso, S.P. Mahulikar, Experimental verification of the role of Brinkman number in micro-channels using local parameters, *Int. J. Heat Mass Transfer* 43 (2000) 1837–1849.
- [54] J. Koo, C. Kleinstreuer, Viscous dissipation effects in microtubes and microchannels, *Int. J. Heat Mass Transfer* 47 (2004) 3159–3169.
- [55] S.A. Bastanjian, A.G. Merzhanov, S.I. Xudiaeov, On hydrodynamic thermal explosion, *Sov. Phys. Doclady* 163 (1965) 133–136.
- [56] Ja.B. Zel'dovich, G.I. Barenblatt, V.B. Librovich, G.M. Makhviladze, *Mathematical Theory of Combustion and Explosion*, Plenum, New York, 1985.
- [57] L. Frenkel, *Kinetic Theory of Liquids*, Clarendon Press, Oxford, 1946.
- [58] D.A. Frank-Kamenetskii, *Diffusion and Heat Transfer in Chemical Kinetics*, second ed., Plenum, New York, 1969.
- [59] N.B. Vargaftik, Y.K. Vinogradov, V.S. Vargin, *Handbook of Physical Properties of Liquids and Gases. Pure Substances and Mixtures*, third augmented and revised ed., Begell House, New York, 1996.